Context effects produced by question orders reveal quantum nature of human judgments

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The hypothesis that human reasoning obeys the laws of quantum rather than classical probability has been used in recent years to explain a variety of seemingly "irrational" judgment and decision-making findings. This article provides independent evidence for this hypothesis based on an a priori prediction, called the quantum question (QQ) equality, concerning the effect of asking attitude questions successively in different orders. We empirically evaluated the predicted QQ equality using 70 national representative surveys and two laboratory experiments that manipulated question orders. Each national study contained 651–3,006 participants. The results provided strong support for the predicted QQ equality. These findings suggest that quantum probability theory, initially invented to explain noncommutativity of measurements in physics, provides a simple account for a surprising regularity regarding measurement order effects in social and behavioral science.

attitude judgment | national surveys | quantum theory | measurement effects

Understanding human reasoning under uncertainty is fundamental for improving decisions about environmental policies, economic planning, public health, and many other important areas. Fifty years of behavioral decision-making research has established that humans do not always follow the "rational" rules of Bayesian probability theory (1). Recently, a group of psychologists and physicists have formulated new rules for human reasoning under uncertainty based on quantum probability theory (2–4). This article reports a test of this theory based on results from a quite different paradigm. We show that the theory implies an a priori and precise prediction called the quantum question (QQ) equality (5). This parameter-free prediction concerns the effect of question order on human judgments. The prediction was tested with the results of 70 national representative surveys, most containing more than 1,000 participants per survey, and two laboratory studies, that manipulated question order. This article presents the QQ equality, its surprisingly strong empirical support, and the key quantum principle, called the law of reciprocity, upon which the QQ equality was mathematically derived. Finally, we explain why human judgments follow quantum rules even if the brain may not be a quantum computer.

The QQ Equality

To introduce the QQ equality, consider three examples of context effects on answers to attitude questions in surveys, illustrated in Table 1. These are the results of three Gallup polls reported in a seminal article on question order effects (6). Each poll included a representative sample of around 1,000 US adults. The participants in one random half of the sample were asked two questions in one order, and those in the other half were asked the same two questions in the opposite order. In the first poll, people were asked whether Bill Clinton was honest and trustworthy, and whether Al Gore was honest and trustworthy. In the second poll, people were asked whether white people dislike black people, and whether black people dislike white people. In the third poll, people were asked whether or not Pete Rose should be admitted to the baseball hall of fame, and whether or not shoeless Joe Jackson should be admitted to the baseball hall of fame. Each column of three two-way tables presents the results from one of the three polls. The cells within the top two tables in each column show the observed proportions for the four response combinations for each question order. For each poll, a "context effect" produced by the question order occurs when any of the four proportions in the top table differs from its corresponding cell in the middle table; these differences are shown in the bottom table. A rigorous statistical test of the four context effects for a single poll can be measured using a χ^2 statistic (*SI Text*). As shown by the χ^2 statistic on the order effects, all three polls produced large and statistically significant ($p < 0.05$) order effects, but with strikingly different patterns. [Note that these order effects go against the commutative property of joint probability: $p(\text{yes } A \cap \text{yes } B) = p(\text{yes } A) \cdot p(\text{yes } B|\text{yes } A) = p(\text{yes } B) \cdot p(\text{yes } A|\text{yes } B) = p(\text{yes } B \cap \text{yes } A)$.]

Despite the different patterns of context effects displayed in the first two polls in Table 1, they both reveal an interesting common property: The sum of the context effects across the two cells within a common diagonal is close to zero. We call this sum the q value. (It is a mathematical property of any context effect table that the q value produced by the main diagonal is always equal but opposite in sign to the q value produced by the off-diagonal; *SI Text*). Our quantum theory, presented later, predicts that the expectation of the q value equals zero for these two polls, $E(q) = 0$, which we call the "QQ equality." For example, for the first poll, the q value computed by summing the context effects for the two off-diagonal cells equals –0.003; the corresponding q value for the second poll equals –0.020. Psychologically, this

Significance

In recent years, quantum probability theory has been used to explain a range of seemingly irrational human decision-making behaviors. The quantum models generally outperform traditional models in fitting human data, but both modeling approaches require optimizing parameter values. However, quantum theory makes a universal, nonparametric prediction for differing outcomes when two successive questions (e.g., attitude judgments) are asked in different orders. Quite remarkably, this prediction was strongly upheld in 70 national surveys carried out over the last decade (and in two laboratory experiments) and is not one derivable by any known cognitive constraints. The findings lend strong support to the idea that human decision making may be based on quantum probability.

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PSYCHOLOGICAL AND COGNITIVE SCIENCES

Table 1. χ^2 results for three Gallup survey experiments reported in a seminal article on question order effects (6)

Observed proportions in the two question orders

	Clinton–Gore			White–black			Rose–Jackson	
	Gy	Gn		By	Bn		Jy	Jn
Cy	0.4899	0.0447	Wy	0.3987	0.0174	Ry	0.3379	0.3241
Cn	0.1767	0.2886	Wn	0.1612	0.4227	Rn	0.0178	0.3202
	Gore–Clinton			Black–white			Jackson–Rose	
	Gy	Gn		By	Bn		Jy	Jn
Cy	0.5625	0.0255	Wy	0.4012	0.1379	Ry	0.4156	0.1234
Cn	0.1991	0.2130	Wn	0.0597	0.4012	Rn	0.0671	0.3939
	Context effects			Context effects			Context effects	
	Gy	Gn		By	Bn		Jy	Jn
Cy	−0.0726	0.0192	Wy	−0.0025	−0.1205	Ry	−0.0777	0.2007
Cn	−0.0224	0.0756	Wn	0.1015	0.0215	Rn	−0.0493	−0.0737
				Test order effects				
χ^2 (3) = 10.14, $p < 0.05$			χ^2 (3) = 73.04, $p < 0.001$			χ^2 (3) = 67.19, $p < 0.001$		
				Test QQ equality				
$q = -0.003$, χ^2 (1) = 0.01, $p = 0.91$			$q = -0.02$, χ^2 (1) = 0.56, $p = 0.46$			$q = 0.1514$, χ^2 (1) = 28.57, $p < 0.001$		

Each column presents the results from one survey. In each column, the top two-way table shows the observed proportions from one question order, the middle two-way table shows those from the other question order, and the bottom two-way table summarizes the context effects. Context effects were computed by subtracting the observed response proportion in each cell obtained in the AB (e.g., Clinton–Gore) order by that in the BA (e.g., Gore–Clinton) order.

means that the number of people who switch from "yes–yes" to "no–no" must be offset by the number who switch in the opposite direction; likewise, the number of people who switch from "yes–no" to "no–yes" must be offset by the number who switch in the opposite direction. (The QQ equality is predicted to hold no matter whether context/order effects occur on neither, one, or both diagonals.) To our knowledge, no traditional psychology theories impose this precise kind of symmetry constraint on context effects.

There is no mathematical constraint that forces the q value to be zero. [In general, the set of all context effects forms a three-dimensional pyramid, but those that satisfy the QQ equality forms only a triangular plane that lies within the pyramid (*SI Text*).] For example, the third poll was selected to be presented in Table 1 because our quantum model predicts that it should produce a violation of the QQ equality, and indeed, the q value is big and equals −0.15. As discussed later, a key assumption required for the derivation of the QQ equality is that the questions being examined are asked successively with no additional information inserted before or between questions. In our examples in Table 1, the first two polls satisfied this condition, but the third did not. A rigorous statistical test of the QQ equality can be computed using a χ^2 test, which we call the q test (*SI Text*). As shown in Table 1, as predicted by our quantum theory, the q tests for the first two polls are not statistically different from zero, but that for the third poll is.

To assess the generality of the QQ equality, we obtained a total of 70 national representative surveys that manipulated the order of questions when they were asked successively. Most of them included more than 1,000 respondents. In addition, we included two laboratory experiments by the authors, each including more than 100 respondents (*SI Text*). Of the 70 national surveys, two of them were the polls shown in Table 1 (the third poll in Table 1 was excluded for the reason mentioned above); a third was another Gallop poll reported by Moore (6); and a fourth was a classic study on context effects (7) that provided the complete two-way tables required for the q test. (Unfortunately, most studies on context effects only report the marginal proportions rather than the complete two-way table required for conducting the q test. The remaining 66 national surveys were all of the available studies conducted by Pew Research Center on various topics during the decade of 2001–2011 that manipulated the order of two questions. (See *SI Text* for more details about the 72 studies.)

The surprising nature of the results concerning the QQ equality can be illustrated in several ways. Because the QQ equality predicts that the sum of the two diagonal entries is zero, it is informative to plot one entry against the other. For each study, we selected the diagonal producing the larger "order effect" (defined as the sum of the absolute values of the two diagonal entries). For example, as seen in Table 1, the larger order effect for the Clinton–Gore poll occurs on the main diagonal, which produced the pair of context effects $x = -0.0726$ and $y = 0.0756$, and the size of the order effect for this pair equals 0.1482. For the white–black poll, the larger order effect occurs on the minor diagonal, which produced the pair of context effects $x = -0.1205$ and $y = 0.1015$, and the size of the order effect for this pair equals 0.2220. Each of the 72 points in Fig. 1, *Left* plots these two (x, y) values from a dataset: The horizontal axis represents the x context effect, and the vertical axis represents the y context effect. More extreme order effects produce more extreme values on the x, y axes. If we ignore the QQ equality, there should not be any a priori constraints on the relations between the pairs, and hence, there is no reason to expect any particular correlation between them. However, according to the QQ equality, the context effect in one cell of a diagonal should be exactly the negative of the context effect in the other cell within the same diagonal, and thus all these pairs should fall along a line with the intercept of 0 and the slope of −1, producing a perfect negative correlation. Fig. 1, *Left* shows the scatter plot of the 72 pairs of context effects from all 72 studies. The straight line in the figure is not a fitted regression line; instead, it is the a priori predicted line with the intercept of 0 and the slope of −1. As shown, the data points fall closely along this predicted line, and the correlation $r = -0.82$ ($r = -0.73$, when two extreme values are excluded). The surprising regularity illustrated in this scatter plot provides strong support for the QQ equality prediction.

Another question concerns the possible range of q values. The finding that the q value remains close to zero is interesting only if the range of its possible values is much larger than the observed q values. The third example of Rose–Jackson poll in Table 1 demonstrates that a large q value can occur, but it is important to

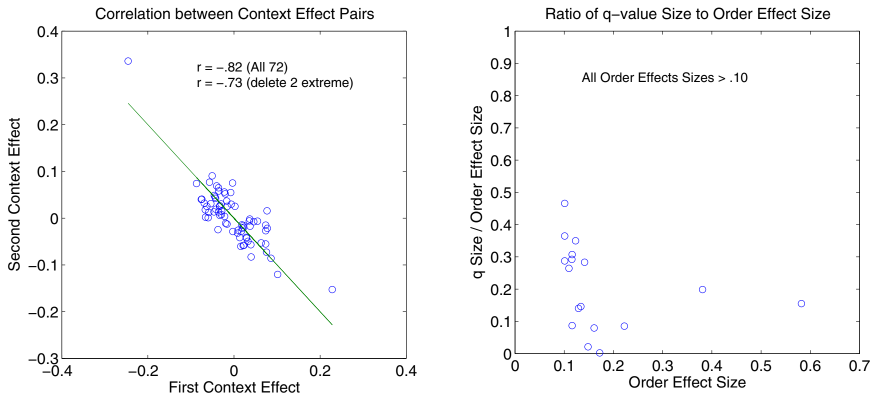

Fig. 1. Empirical demonstration of the QQ equality. *Left* shows the context effect from one diagonal cell plotted against the context effect from the other cell within the same diagonal. The QQ equality is satisfied when the data points fall on the predicted line with the intercept of 0 and the slope of −1 (shown here). *Right* shows the ratio (*q* value size/order effect size) plotted as a function of the size of the order effect, and the QQ equality predicts that, when order effects are large enough to emerge from the noise, then this ratio should start below 1.0 and go to zero as the order effect increases.

examine this issue for all 72 studies. For any observed table of context effects, we can bound the *q* value by the size of the order effect. Recall that we defined the size of the order effect in terms of the diagonal with the larger summed absolute values of context effects (e.g., 0.15 for Clinton–Gore and 0.22 for white–black). The *q* value can possibly equal but cannot exceed the size of the order effect defined in this manner (*SI Text*). If the order effect is close to zero, then the *q* value must also be close to zero, and sampling estimation error for both will cause them to be approximately equal in size, so the *q* test is only interesting when the order effects are well above zero. The relation between the *q* value and the order effect can be described by their ratio: size of the *q* value/size of the order effect. Because of the sampling error, this ratio will necessarily be close to 1 when the order effect is very small, but if the QQ equality holds, then this ratio should drop to zero as the size of the order effect increases. Fig. 1, *Right* plots this ratio for the 17 studies that produced an order effect greater than 0.10. As predicted by the QQ equality and shown in the figure, this ratio starts well below 1.0 when the order effect is small and drops toward zero when the order effect becomes large; over the entire range, the *q* value remains small.

Fig. 1 may provide a compelling illustration, but it does not substitute for an appropriate statistical test of the null hypothesis that the expectation of the *q* values is zero. We exclude the four national surveys that were specifically selected from previous studies because they found question order effects (although including them does not change our conclusions below; *SI Text*); and we analyze the distributions of χ^2 statistics for order effects and *q* values from the remaining 66 Pew surveys that were selected without any bias for either test. As described before, these include all of the datasets available from Pew in the past decade that manipulated the order of two questions. On the one hand, the χ^2 distribution test for order effects produced a significant deviation from the null hypothesis ($p = 0.0004$); on the other hand, the χ^2 distribution test for the *q* values indicates no significant deviation from the null hypothesis ($p = 0.4625$) (see *SI Text* for detail on the χ^2 tests). Taken together, these results show that across all 66 Pew datasets, there are significant question order effects, and the QQ equality holds as predicted.

In summary, we have presented strong evidence that context effects produced by the order of questions satisfy the QQ equality predicted by quantum theory: (*i*) The context effect from one cell of a diagonal is negatively correlated with that from the other cell; (*ii*) the *q* value remains small even as the size of the context/order effect increases; (*iii*) the *q* values do not differ significantly from zero as tested by a large set of national survey data on various topics collected in the past decade. We do not know of any existing cognitive constraints that would produce these symmetrical results for context effects. It is possible to construct a model that is narrowly constrained to satisfy the QQ equality, but these constraints could also prevent the model from accounting for order effects (see *SI Text* for two such examples, one based on a model that assumes a probability of repeating the first choice, and another that assumes an anchoring-adjustment process). What is needed is a general theory for question order effects that satisfies the QQ equality constraint. We hope that these findings prompt researchers to look for alternative accounts. In any event, we turn next to the basis for the quantum theory prediction.

Quantum Model of Measurement Order Effects

The discovery of the QQ equality was not an accident. This law was predicted a priori from a quantum probability model of human judgment (5). The model is simple and intuitive, and the derivation for the test is general and parameter free. We begin with a cognitive-process interpretation of the theory and later present it formally. (See ref. 3 for a general introduction to quantum probability applied to cognition and decision.)

The general idea may be stated in the following way. The knowledge that a person has and uses to answer questions can be represented as a very high multidimensional space, *H*. This space can be described by a set of orthogonal axes (technically termed "basis vectors" below) that is chosen to answer the questions. Many cognitive theories represent knowledge as a vector of feature values, and one can think of the axes in these terms. For example, if features are binary and there are 100 relevant features, then each of the 2^{100} axes can be used to represent a different pattern of ones and zeros where a 1 represents presence and a zero represents absence of that feature (e.g., ref. 8). A person's beliefs about events are represented by a unit length vector, generally at an oblique angle with respect to these axes. The projection of the belief vector onto an axis can be described as a belief that a feature is present. *H* does not change with the question asked or with the context in which the question occurs, but the way the knowledge in *H* is used changes with both factors. Of course, not all of the knowledge in *H* is needed to answer a given question, and the knowledge that is to be considered for answering a given question, A, is a subspace, S_A, of *H*. The knowledge used to answer another question, B, is represented by another subspace, S_B, which generally is of different dimensionality and is not necessarily aligned with the axes chosen to describe S_A. For example, if *H* is represented by a cube and S_A is the square plane on the bottom of the cube, then S_B could be another plane containing the cube's major diagonal. Finally, the probability of affirming an answer is determined by the square of

PSYCHOLOGICAL AND COGNITIVE SCIENCES

the projection of the current belief vector onto the subspace used to answer the question. [One might question the necessity of computing probabilities based on squared length of projections; however, Gleason's theorem (9) proves that this is the only way to assign an additive probability measure to all subspaces of dimension greater than 2.]

It is useful to make this abstract description concrete by considering the following toy example. Imagine a survey respondent who is asked questions about Bill Clinton during the period when Clinton was the US president, such as "Is Clinton a respectable leader?" and "Is Clinton doing a good job as the president?". Suppose for simplicity that only two binary features are used by the respondent to answer these questions: The economy is doing well (yes = 1, no = 0), and a leader should exhibit marital fidelity (yes = 1, no = 0). The four combinations, (11, 10, 01, 00), form a four-dimensional space *H*, spanned by four basis vectors, one for each combination. It is difficult to visualize a four-dimensional space, and so we will assume that nonzero beliefs (that is, 11, 10, and 01) are assigned by the respondent only to the first three of the four basis vectors and zero (that is, 00) is assigned to the fourth basis vector. In Fig. 2, the first three basis vectors are labeled *X* (corresponding to 11), *Y* (corresponding to 10), and *Z* (corresponding to 01). Assume that the answer "yes" to the question "Is Clinton doing a good job as the president?" is satisfied by the property "the economy is doing well," and so it is represented by the plane *C* defined by the axes *X* and *Y*. The orthogonal axis *Z* represents the answer "no" to this Clinton question. How these subspaces are used depends on the present context, including the way the person is thinking at the time, which determines the weighting on pieces of information in the subspace. The current context and current way of thinking is represented as a "belief vector" of unit length, denoted here as *S*, located at some orientation with respect to the *X*, *Y*, *Z* axes. One can think of this current belief as defined by the current contents in the person's short-term memory. In Fig. 2, the initial belief state *S* is the largest on the 01 combination *Z*. As illustrated in Fig. 2, *Upper*, the probability of answering "yes" to the question "Is Clinton doing a good job?" is obtained by projecting the current belief vector *S* down onto the subspace *C*, and squaring its length (which equals 0.33 in this example).

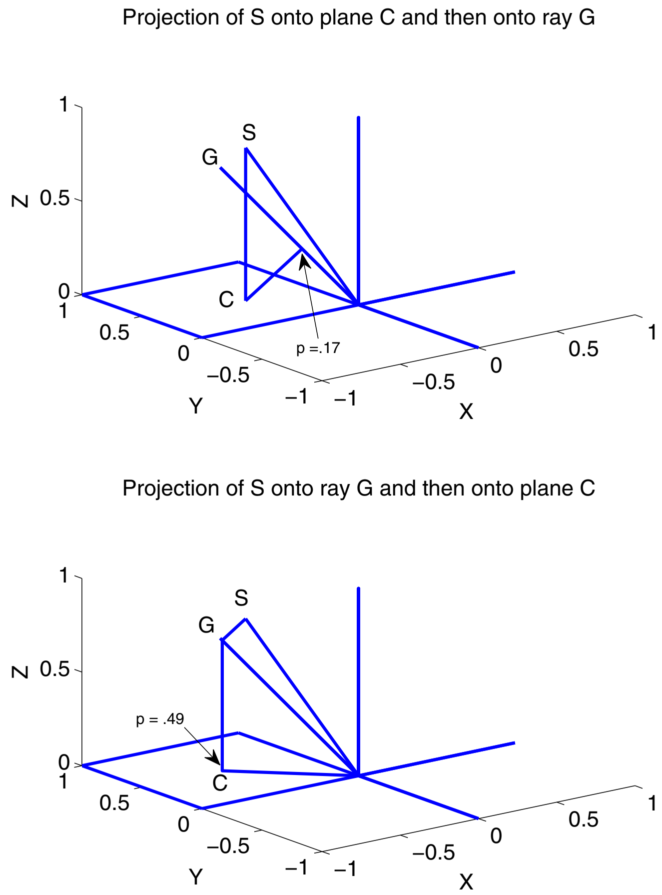

Fig. 2. An illustration of basic quantum principles used in the measurement order model. The figure illustrates a simple three-dimensional vector space, spanned by basis vectors *X*, *Y*, *Z*. The vector *S* represents the initial belief state. The squared length of first projecting *S* on the plane *C* and then projecting on the ray *G* equals 0.17; the squared length of first projecting on the ray *G* and then on the plane *C* equals 0.49. See text for an example instantiating these terms.

Imagine a similar question is asked about Al Gore: "Would Gore be a good next president instead?" Suppose the answer "yes" to this Gore question is represented by the one-dimensional subspace spanned by the vector *G* in Fig. 2, which lies at an oblique angle with respect to the subspace *C*. Here, as illustrated by the Clinton and the Gore questions, subspaces used to answer different questions can have different dimensionality and at different orientations with respect to each other. Note that the subspace for the answer "yes" to the Gore question is a ray that is not contained in *C* (economy is doing well), and it is not aligned with *Z* (economy is not doing well) either. Psychologically, this represents the idea that the person prefers the answer "yes" to the Gore question when there is uncertainty about the state of the economy. As illustrated in Fig. 2, *Lower*, the probability of answering "yes" to the Gore question is obtained by projecting the current belief vector *S* onto *G* and squaring its length (which equals 0.97 in this example).

Now we come to the heart of the model: context and order effects. As time passes and new information arrives, the content of short-term memory changes, and the belief vector changes accordingly. When two questions immediately follow each other, then after answering the first question, the belief vector that was used to answer the first question changes to match the answer just given. In other words, the belief vector realigns with the current contents of short-term memory (which includes answers to previous questions) and the perspectives that flow from those contents. In geometric terms, the new belief vector used for the second question is simply the projection of the initial belief vector onto the subspace used to answer the first question, normalized to have unit length. This new belief vector is then projected onto the subspace used for answering the second question, and squared to produce the probability of a "yes" response to the second question. Using this process, the probability of the sequence of "yes" answers equals the squared length of the projection produced by first projecting the belief state onto the subspace for answering "yes" to the first question, and then projecting the updated belief vector onto the subspace for answering "yes" to the second question. When the two subspaces lie at oblique angles with respect to each other, the order of answering the questions will change the projections and ultimately the probabilities of the responses, and this is described as "noncommutativity" in quantum theory. This is where a context effect arises.

This process is illustrated in Fig. 2 for the case in which the answers to both questions are "yes." Fig. 2, *Upper* shows the process when the Clinton question is asked before the Gore question. The initial state *S* is first projected onto the plane *C* and then projected onto the ray *G*, and the squared length of the final projection equals 0.17, which gives the probability of "yes" to Clinton and then "yes" to Gore. Fig. 2, *Lower* shows the process when the Gore question is asked before the Clinton question. The initial state S is first projected onto the ray G and then projected onto the plane C, and the squared length of the final projection equals 0.49, which gives the probability of "yes" to Gore and then "yes" to Clinton.

It should be noted that not all subspace combinations will produce context/order effects. When the two subspaces are not at oblique angles, the order of answering the questions (that is, projecting the belief vector to the subspaces) does not matter. In the example above, it is assumed that the pair of subspaces "the economy is doing well" and "a leader should exhibit marital fidelity" do not produce context/order effects. Such cases are described as "compatible" in quantum theory. In comparison, the cases with order effects are described as "incompatible." A psychological theory could be developed to predict which question combinations would be compatible or incompatible, but that is beyond the scope of the present article (5). Such a theory is not needed for the present purposes because the QQ equality prediction (discussed next) holds for both compatible and incompatible situations. If the two subspaces are compatible, then there are no context effects, no order effects, and the QQ equality holds for trivial reasons.

Up to this point, we have merely presented a "geometric" model description of the way that the quantum formulation will produce context/order effects. The existence of such effects might be explained by any number of cognitive theories and processes. However, the quantum formulation predicts a particular relation among the observed context/order effects, the one described as the QQ equality in the first part of this article. This prediction follows from the "law of reciprocity" often discussed in quantum theory (e.g., ref. 10, p. 34). The essential idea is that the transition from one state to another depends only on the correlation between the states as measured by their inner product. This prediction will be laid out in the formal description to follow.

Now consider the general problem of computing answers to a sequence of questions. Recall that each answer to a question is represented by a subspace (e.g., a ray, a plane, a hyperplane) in an N-dimensional vector space, and so a pair of answers to questions is represented by two different subspaces. Each subspace corresponds to a "projector" that projects state vectors onto the subspace. The probability of agreeing to question A (denoted Ay and represented by subspace S_A corresponding to projector P_A) and then agreeing to question B (denoted by By and represented by subspace S_B corresponding to projector P_B) equals the squared length of the result obtained by sequentially projecting the prior belief state on the subspace for agreeing to A and then on the subspace for agreeing to B, that is, $p(\text{AyBy}) = \|P_B P_A S\|^2$. The probability of agreeing to a question B and then agreeing to question A equals the squared length of the result obtained by sequentially projecting the prior belief state on the subspace for agreeing to B and then on the subspace for agreeing to A, that is, $p(\text{ByAy}) = \|P_A P_B S\|^2$. If the projectors are commutative (i.e., $P_A P_B = P_B P_A$), then the subspaces are compatible, and no order effects are predicted to occur. If the projectors are noncommutative (i.e., $P_B P_A \neq P_A P_B$), then the subspaces for the two questions are incompatible, and order effects are predicted to occur.

QQ Equality Derived from the Quantum Model

If two questions, adjacent to each other, are asked in different orders, then the quantum model of measurement order as described above makes an a priori and parameter-free prediction, named the QQ equality (see *SI Text* or ref. 5 or ref. 3 for proofs):

$$q = [p(\text{AyBy}) + p(\text{AnBn})] - [p(\text{ByAy}) + p(\text{BnAn})]$$
$$= [p(\text{AyBn}) + p(\text{AnBy})] - [p(\text{ByAn}) + p(\text{BnAy})] = 0.$$

The first line implies that the two main diagonal cells of the context effect table sum to zero, and the second line implies that the two off-diagonal cells in the context effect table sum to zero (see Table 1 for examples). Intuitively, this means, the probability of having the same response to the two questions should remain invariant across the two question orders; also the probability of having different responses to the two questions should remain invariant across the two question orders. This equality must hold even when context effects produced by the question order occur so that, for example, $p(\text{AyBn}) \neq p(\text{BnAy})$ and $p(AyBy) \neq p(ByAy)$. As shown in the proof (*SI Text*), this equality must hold for any initial belief state and any pair of projectors in any high-dimensional vector space. The QQ equality is still predicted even if there are individual differences in the initial belief state S, so that it continues to hold when we average across individuals with different belief states (i.e., a mixed state; *SI Text*). As we have shown, this precise prediction can be easily tested empirically: If it holds, the difference in observed proportions on the left hand of the QQ equality, defined as q, should not statistically differ from zero as tested by a χ^2 test for difference in proportions. As introduced earlier, this q test was indeed satisfied for the large dataset of 66 national representative surveys on various topics.

Why does the quantum model predict the QQ equality? The proof (*SI Text*) is based on a fundamental principle of quantum theory called the "law of reciprocity" (10). The probability of transiting from a projection on subspace S_A to a projection on subspace S_B equals the probability of transiting from a projection on subspace S_B to a projection on subspace S_A. More formally, for any given state vector S and two projectors P_A, P_B, if $T = P_A S$ and $V = P_B S$, then $|\langle T|V\rangle|^2 = |\langle V|T\rangle|^2$. The latter is true even when $P_A P_B \neq P_B P_A$. The derived QQ equality provides a simple way to test this fundamental principle of quantum theory.

A critical assumption underlying the derivation of the QQ equality is that starting from a common state (technically, the state is represented by a density matrix for a mixture of people with individual differences), questions are asked back to back without any information inserted in between so that two successive projections are applied (either $P_A P_B$ or $P_B P_A$) to the common state. New information presented before or inserted in between questions will change the state in different ways. For example, if new information is presented in between questions, then a change is produced by a transformation U' for one type of information, and a different change is produced by another transformation U'' for a different type of information. Rather than comparing $P_A P_B S$ with $P_B P_A S$ (as required for the derivation of the QQ equality), we are now comparing $P_A U'' P_B S$ with $P_B U' P_A S$, and the equality is no longer expected to hold. (For example, the Rose–Jackson poll in Table 1 violated this condition for our q test. A more complex quantum model that includes the transformations U' and U'' is required for this case.)

The QQ equality prediction from our model depends on the assumption that the belief vector used to answer the second question is the normalized projection used to assign probability to the first question. We have looked at deviations from this assumption: If the new belief vector is assumed to be some proportion of the angular distance between the initial belief vector and the resultant projection, the q value is not exactly zero; but in all of the cases we have examined, it is very close to zero (*SI Text*). This suggests that the prediction derived from our quantum model may be even more general than claimed here and may apply to a wider class of cognitive context effects. We leave this possibility open for further research.

Discussion

The surprisingly strong evidence for the QQ equality supports the quantum model of measurement order effects. It is part of an accumulating body of research showing that quantum theory can explain a wide range of behavioral findings that are paradoxical from a classical probability perspective (11, 12). Very likely, the QQ equality is the strongest form of support because its prediction is not dependent on parameter choices—other applications of quantum theory to human cognition depend on choosing parameter values to best fit data (as do most cognitive models).

This support of the quantum probability approach leaves a question: Can a classical brain give rise to behavior that follows quantum principles (13)? Mathematical physicists have recently provided a plausible account, showing that quantum behavior can emerge when coarse measurements of a classical dynamic system generate incompatible observables that result in unresolvable uncertainty relations and entangled correlations (14, 15). Scientists are still far from understanding how mental states (such as judgments and decisions) emerge from the neural substrates. It is too early to conclude whether or not quantum physics plays a significant role in this emergence (16–19). Regardless, even if the brain's neural processes operate by classical rules, quantum probability may provide a better description than classical probability for the way humans reason under uncertainty. Applications of both the QQ equality and a wide class of psychological and decision-making tasks (20–24) support this hypothesis, and they share the following conceptual bases: (*i*) Human judgments, such as attitude judgments, are often not simply read out from memory, but rather, they are constructed from the cognitive state for the question at hand; and (*ii*) drawing a conclusion from one question changes the context and disturbs the cognitive system, which then (*iii*) affects the answer to the next question, producing order effects, so that (*iv*) human judgments do not always obey the commutative rule of Boolean logic. If we replace "human judgments" with "physical measurements" and replace "cognitive system" with "physical system," then these are exactly the same reasons that led physicists to develop quantum theory in the first place. The QQ equality presented in this paper shows that quantum probability theory, used to explain noncommutativity of measurements in physics, provides a strongly supported prediction for measurement order effects in social and behavioral science.

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